PREPROCESSING COMPLEXITY FOR SOME GRAPH PROBLEMS PARAMETERIZED BY STRUCTURAL PARAMETERS

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PREPROCESSING ON GRAPHS USING STRUCTURAL PARAMETERS

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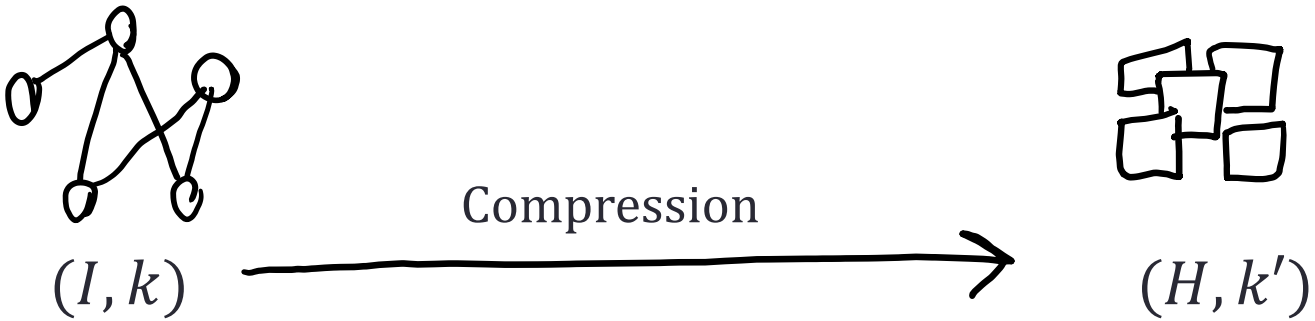
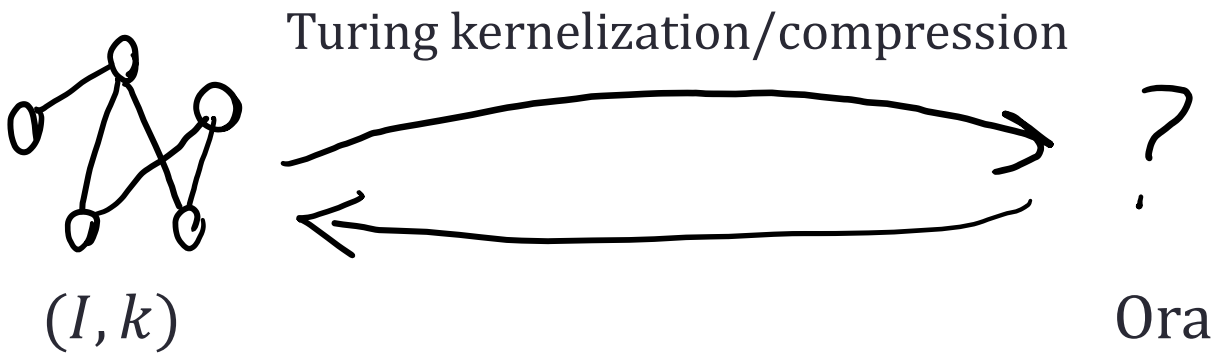
Kernelization
(I, k)
(I', k')


Compression
(I, k)
(H, k')


Turing kernelization/compression
?
(I, k)
Oracle

Kernelization for problem Q

- polytime algorithm on instance (I, k) of Q
- produces instance (I', k') of Q
- $(I, k) \in Q \Leftrightarrow (I', k') \in Q$
- $|I'|, k' \in O(f(k))$ for some function f that depends only on k

Good when $f(k) \in O(k^c)$. If this holds, we have a polynomial kernel.

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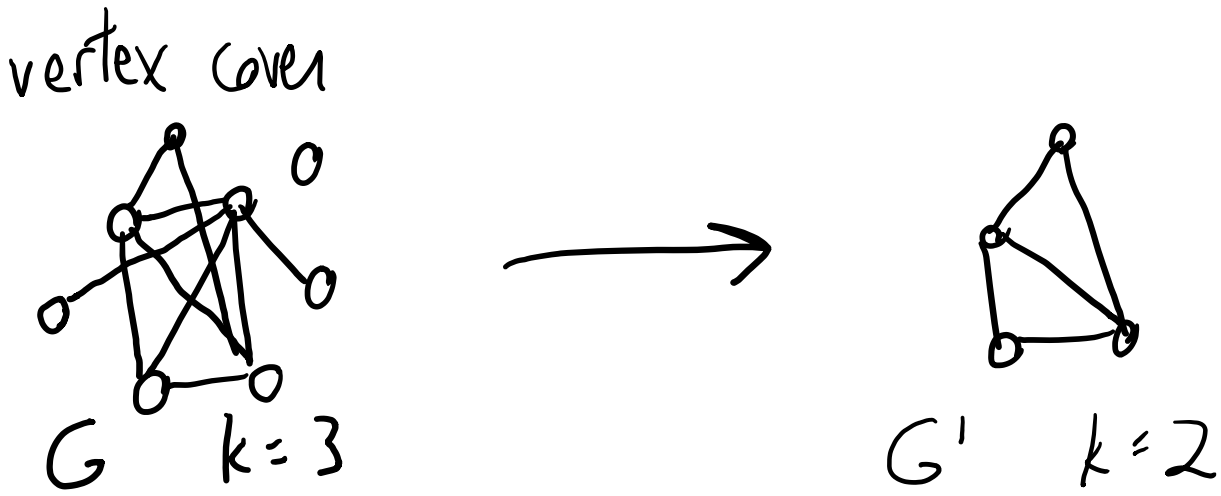
Example:

- Vertex cover problem:
- Given graph G and parameter k
- Does there exist a vertex cover of size at most k?

Reduction rules:

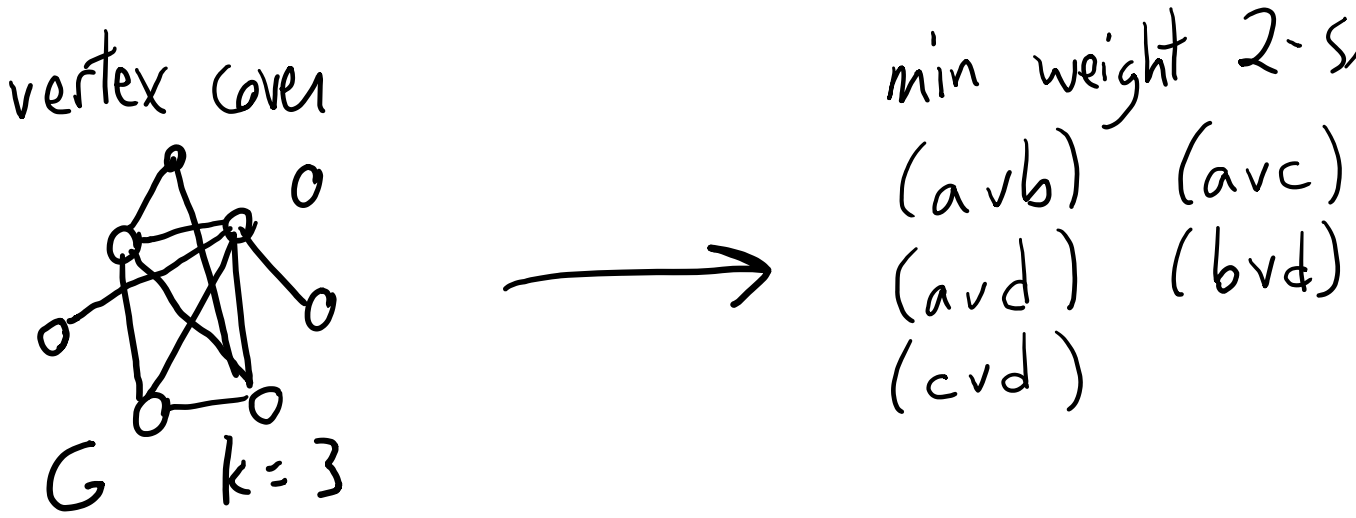
- Delete isolated vertices
- Delete vertices of degree $> k$, reduce k by 1

Preserve an equivalent instance of size $O(k^2)$.



Compression for problem Q

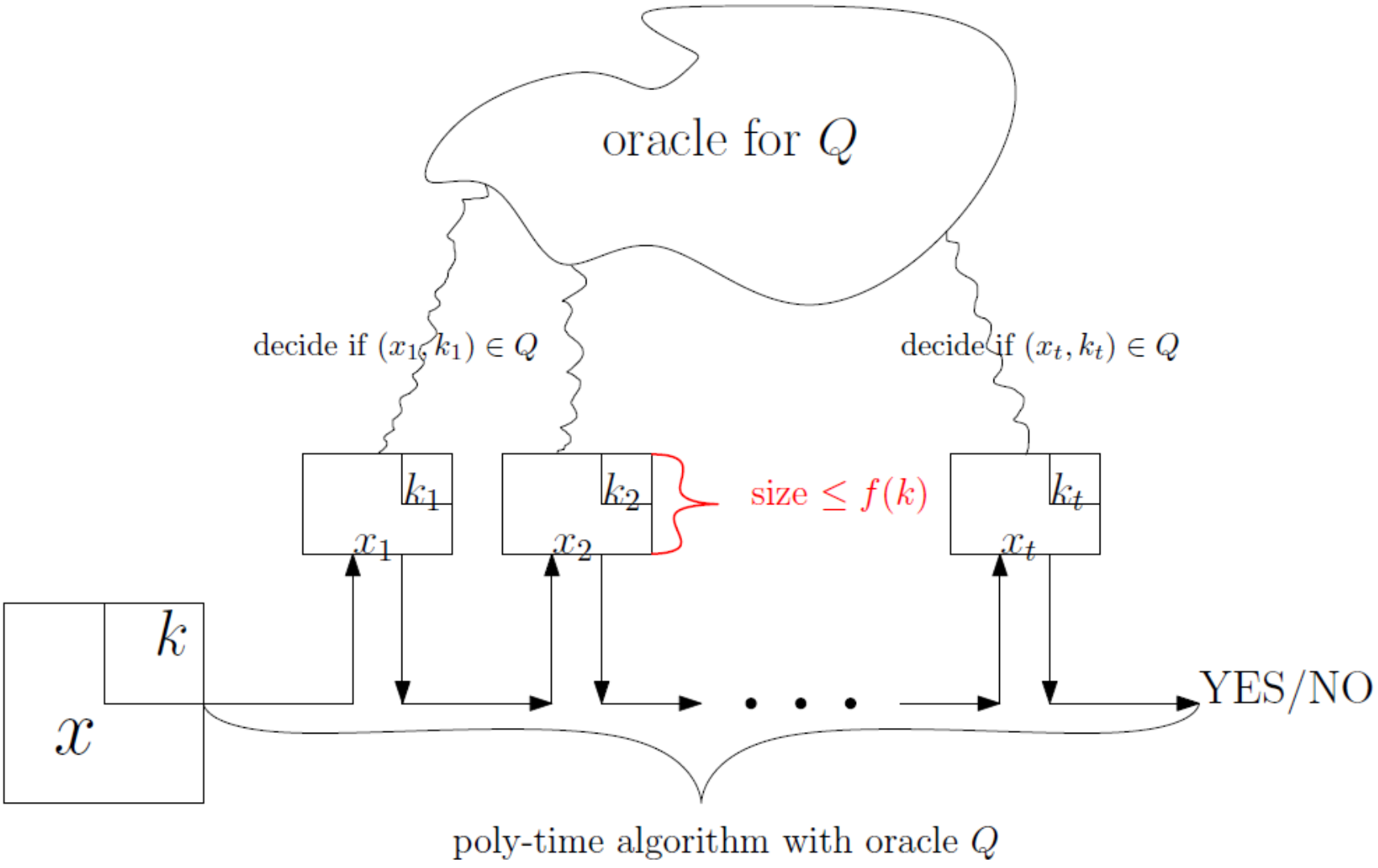
- polytime algorithm on instance (I, k) of Q
- produces instance (I', k') of **some other problem** R
- $(I, k) \in Q \Leftrightarrow (I', k') \in R$
- $|I'|, k' \in O(f(k))$ for some function f that depends only on k



Turing kernelization for problem Q

- polytime algorithm on instance (I, k) of Q
- can query an Oracle O for instances (I', k') of Q where $|I'|, k \in O(f(k))$

If query sizes are $O(k^c)$, we have a polynomial Turing kernel.


oracle for Q
decide if $(x_1, k_1) \in Q$
decide if $(x_t, k_t) \in Q$
k_1
x_1
k_2
x_2
size $\leq f(k)$
k_t
x_t
k
x
YES/NO
poly-time algorithm with oracle Q

Turing compression for problem Q

- polytime algorithm on instance (I, k) of Q
- can query an Oracle O for instances (I', k') of R where $|I'|, k \in O(f(k))$

PK = polynomial kernel

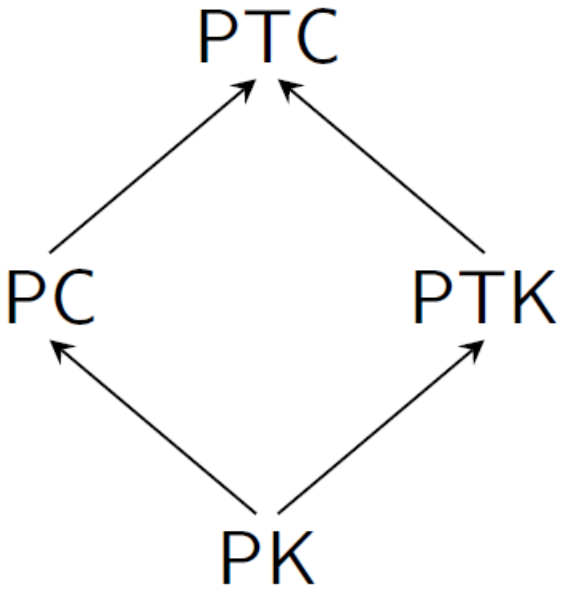
PC = polynomial compression

PTK = polynomial Turing kernel

PTC = polynomial Turing compression

WK-hard = probably no PK

MK-hard = probably no PK



Natural parameter: k = value to optimize.

- Vertex cover admits a polynomial kernel of size $2k$, where k is the size of the cover.
- Clique (probably) does not admit a kernel of any size wrt k, where k is the size of the clique.

Structural parameter: kernelize w.r.t. some graph invariant.

- Max-Clique admits a kernel of size $2^{vc} + vc$, where vc is the size of a vertex cover of G.
- Max-Clique admits a Turing kernel of query size $vc + 1$. (explained later)

In this paper

What is the preprocessing complexity of algorithmic problems, parameterized by:

- vertex cover (vc)
- twin-cover (tc)
- neighborhood diversity (nd)
- modular-width (mw)

For problem Q, we write e.g. $Q(vc)$ to indicate that we parameterize by vc.

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= Weidong

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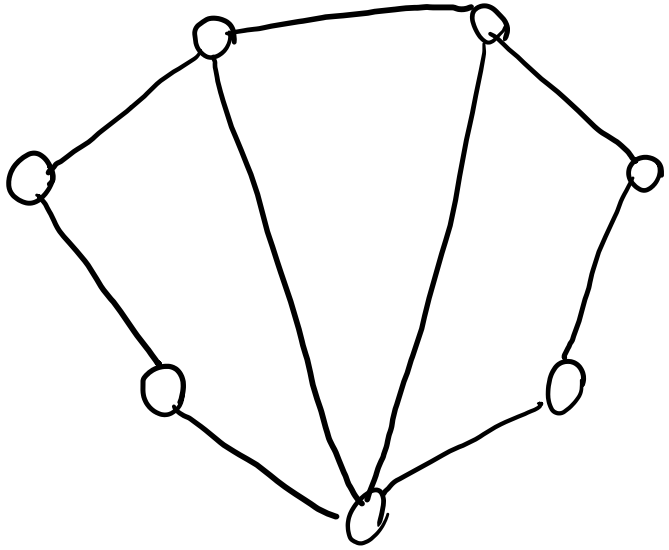
Vertex cover - $vc(G)$ (or just vc)

vc is the minimum size of a subset of vertices that touches every edge.



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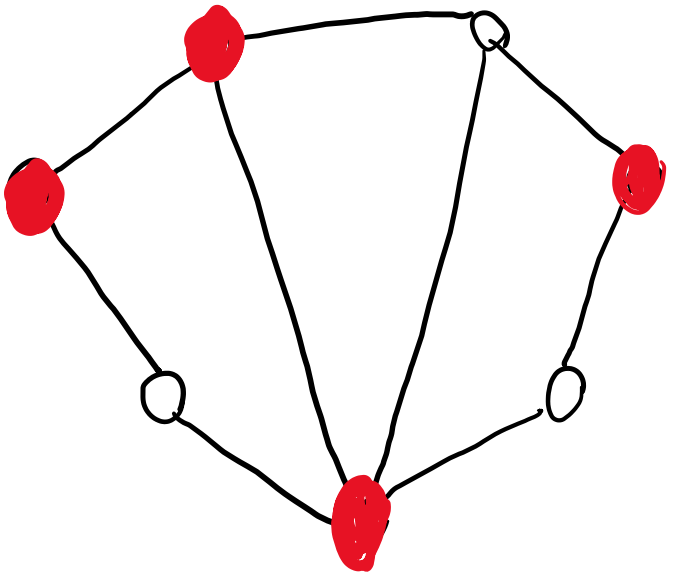

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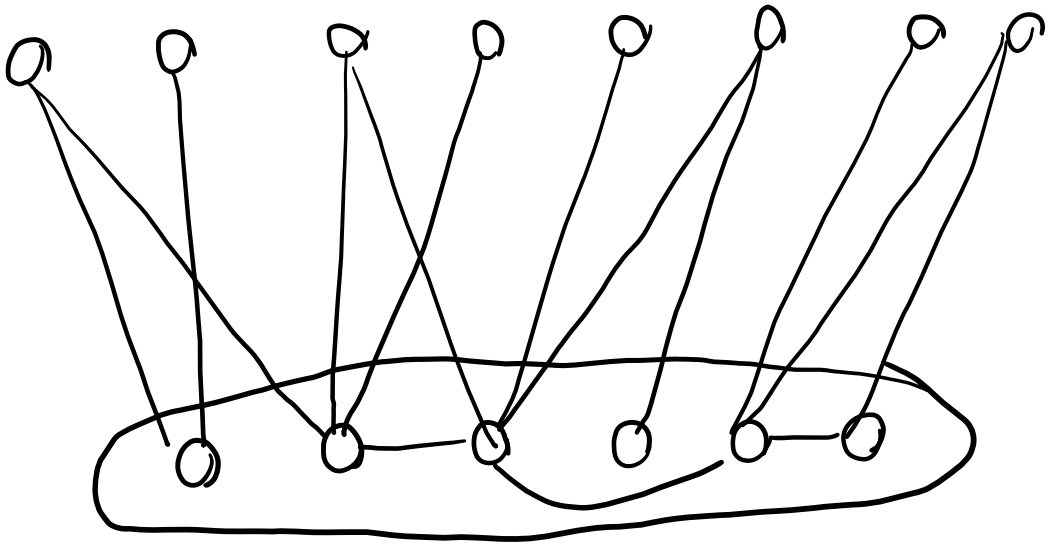
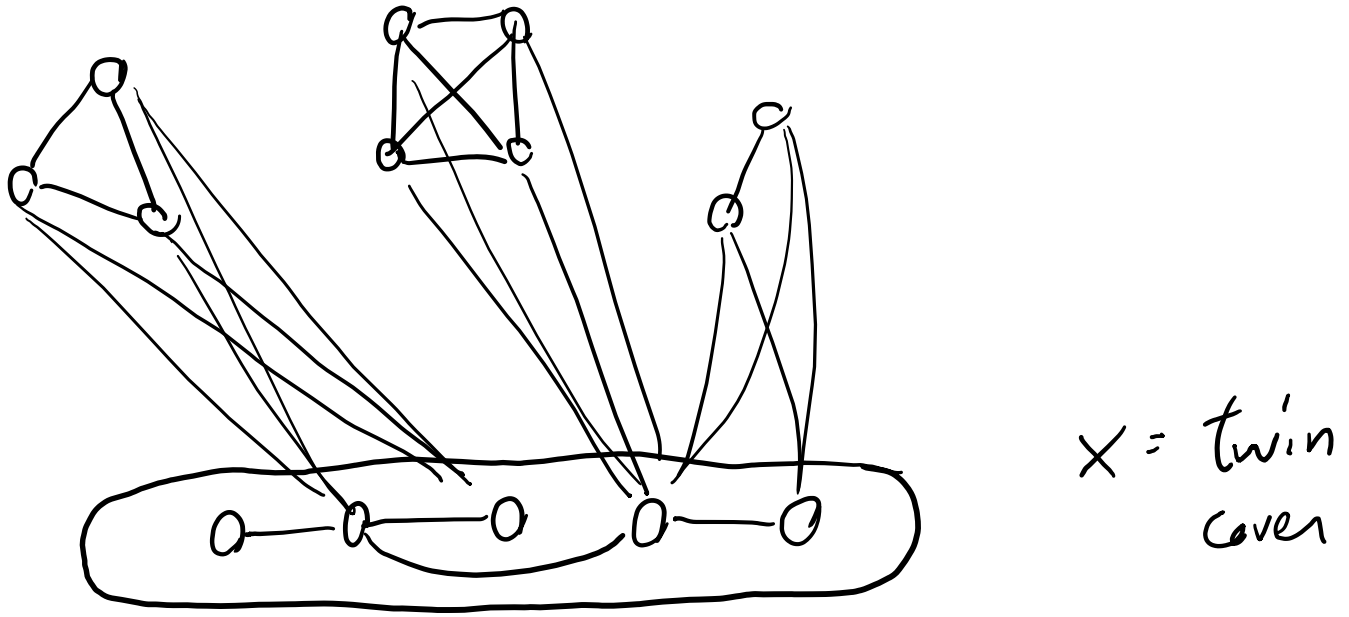


Twin cover - $tc(G)$

Introduced in [Ganian 2011].

tc is the minimum size of a subset $X \subseteq V(G)$ such that $G - X$ is a cluster graph (contains disconnected cliques), and vertices in the same clique of $G - X$ are twins.



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Similar to vc, except that every outside vertex is blown up into a clique.

$tc \leq vc$

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Max-Clique(tc)

- Max-Clique does not admit a polynomial compression in parameter vc unless coNP $\subseteq$ NP$_{/\text{poly}}$ [Bodlaender, Jansen, Kratsch, 2014]
  - No poly kernel in vc nor tc either.

Proposition

Max-Clique admits a polynomial Turing kernel in parameter tc.

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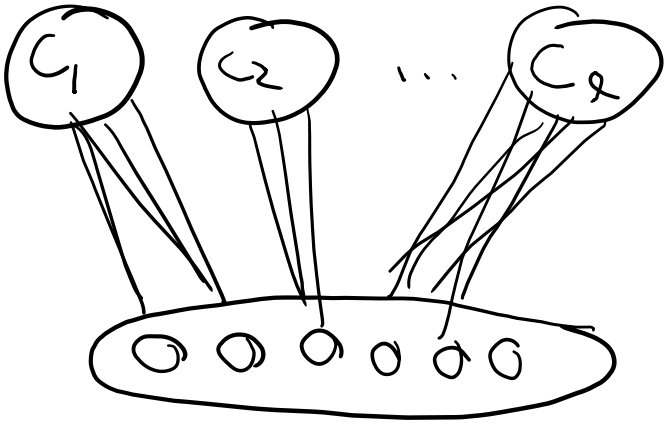
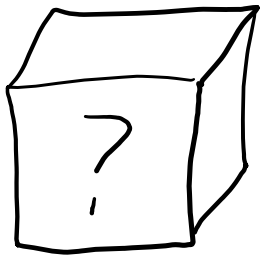
Let X such that $G - X$ is a cluster graph, each clique containing twins. For each clique C in $G - X$:

- Take any $c \in C$, ask Oracle for Max-Clique in $G[X + c]$ (query size $tc + 1$).
- If Oracle returns k, then using C we can have a clique of size $k + |C| - 1$ (some details skipped)





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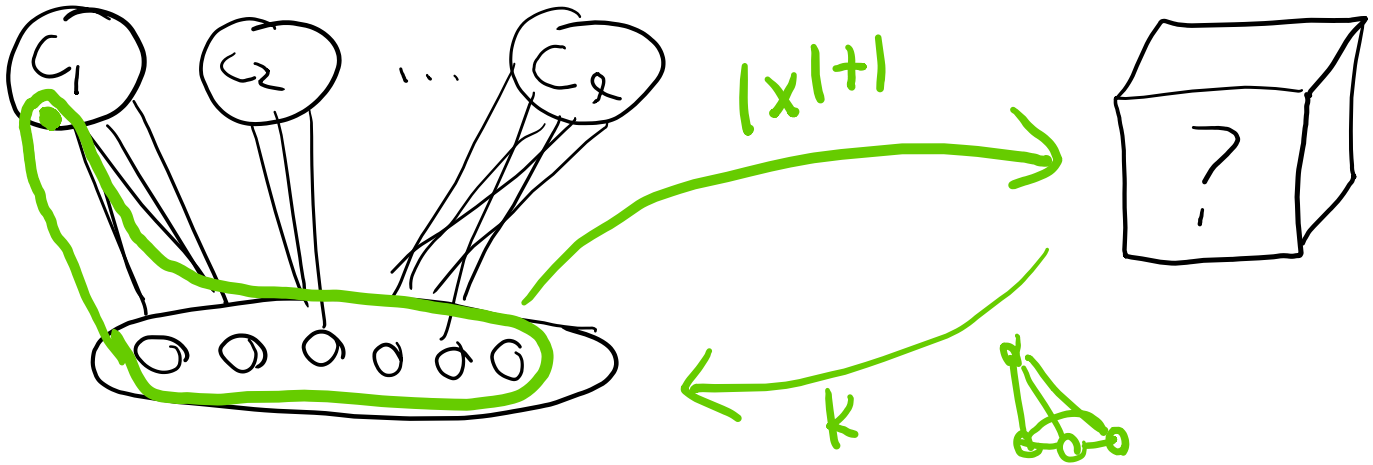

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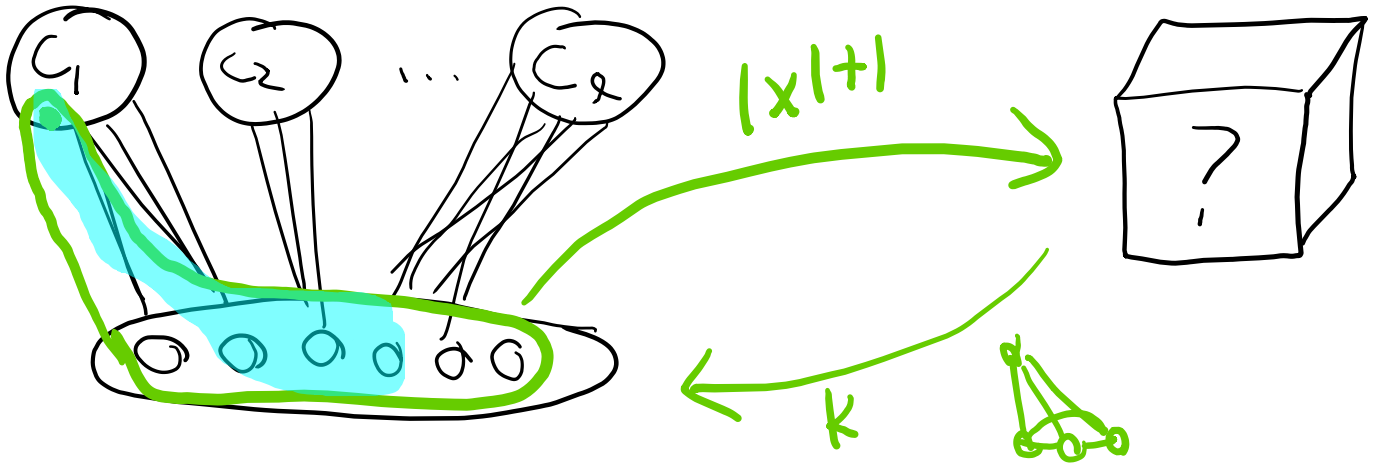

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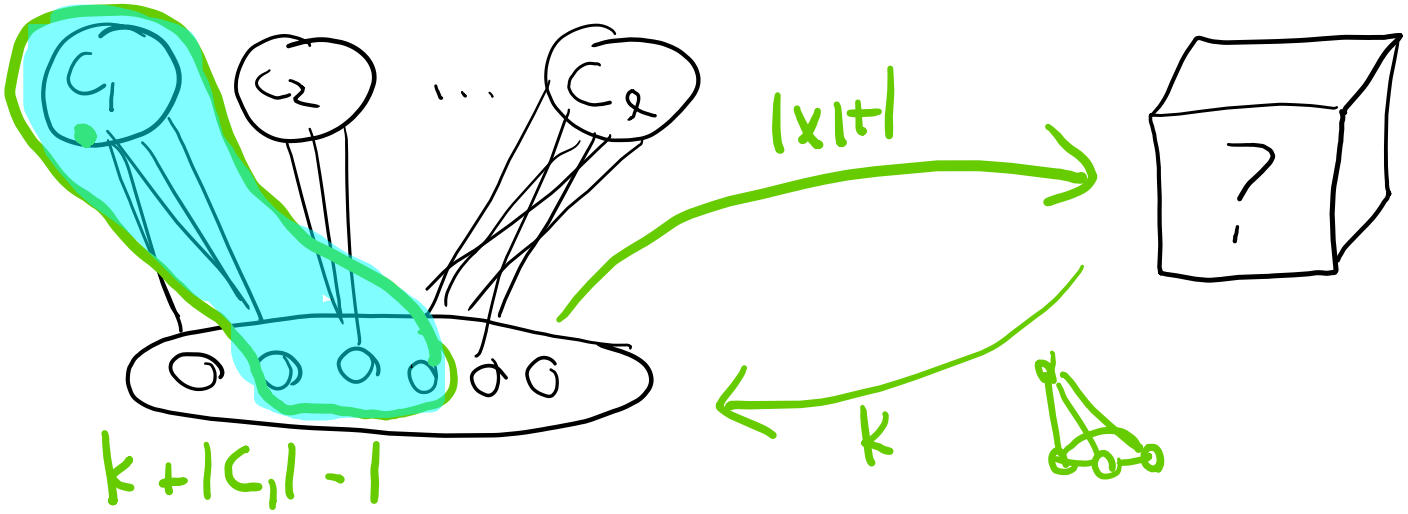

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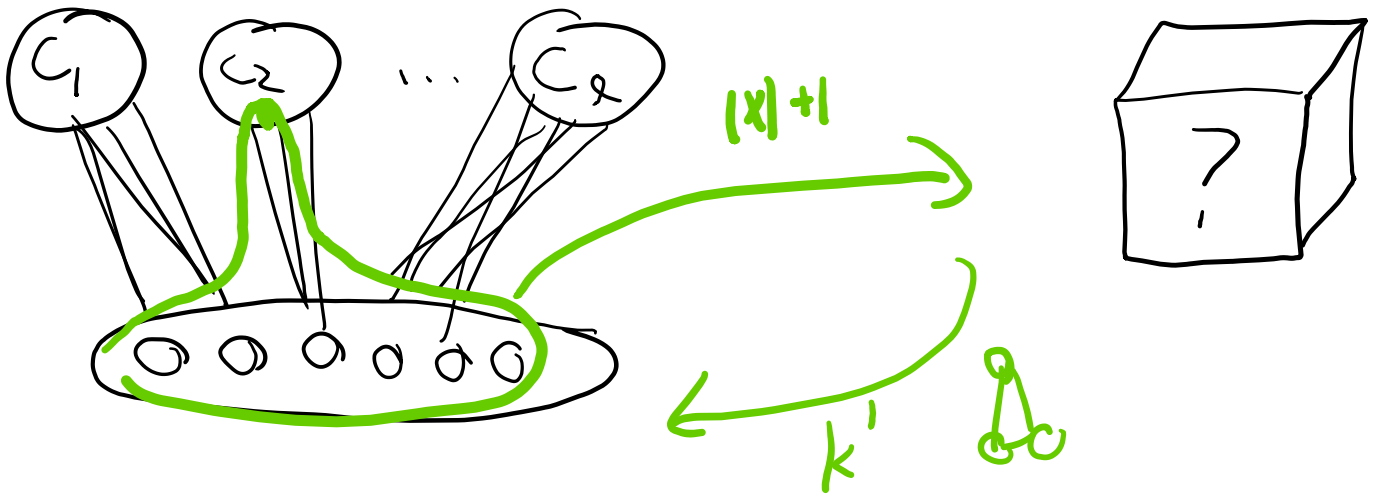

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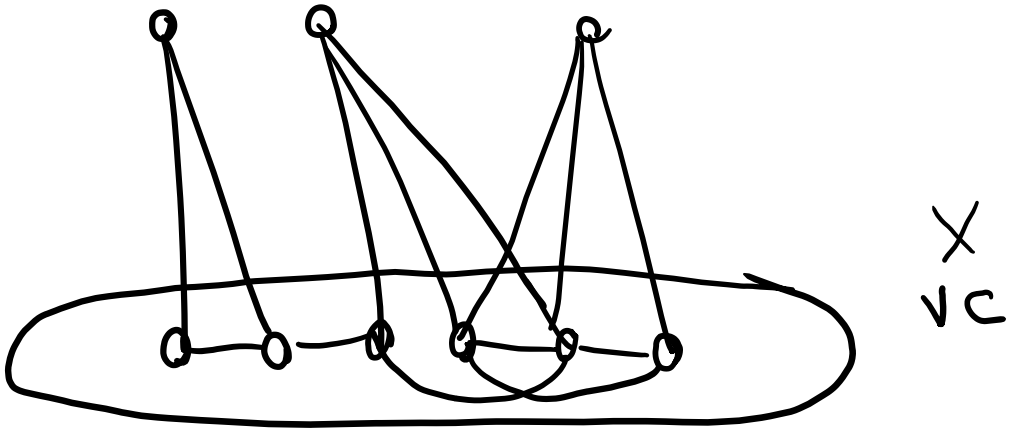
Triangle-Partition

Input: graph G

Output: is there a partition of $V(G)$ into $n/3$ cliques of size 3?

Triangle-Partition(vc): trivial $3/2 \cdot vc$ kernel.



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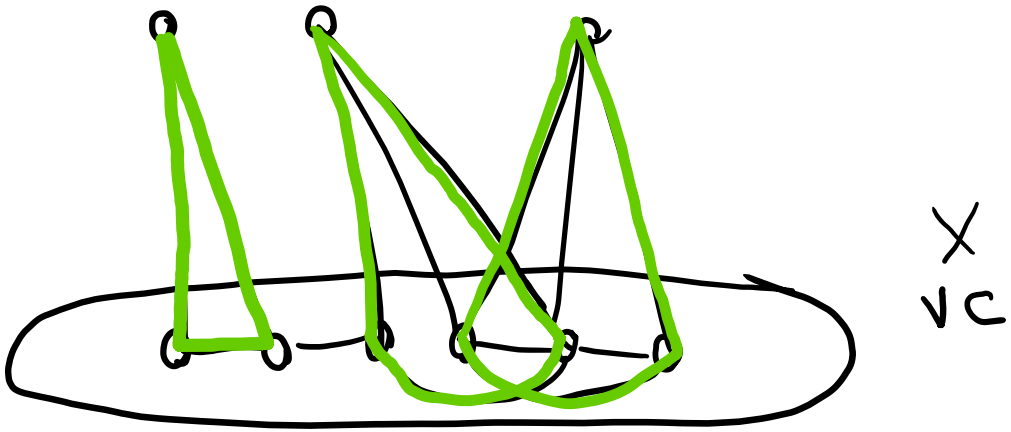

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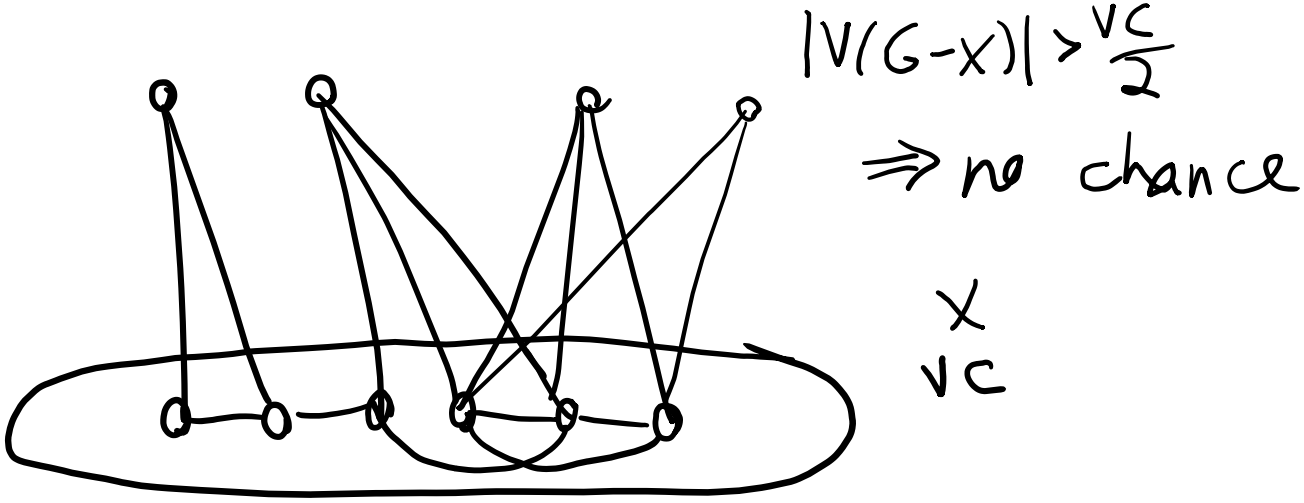
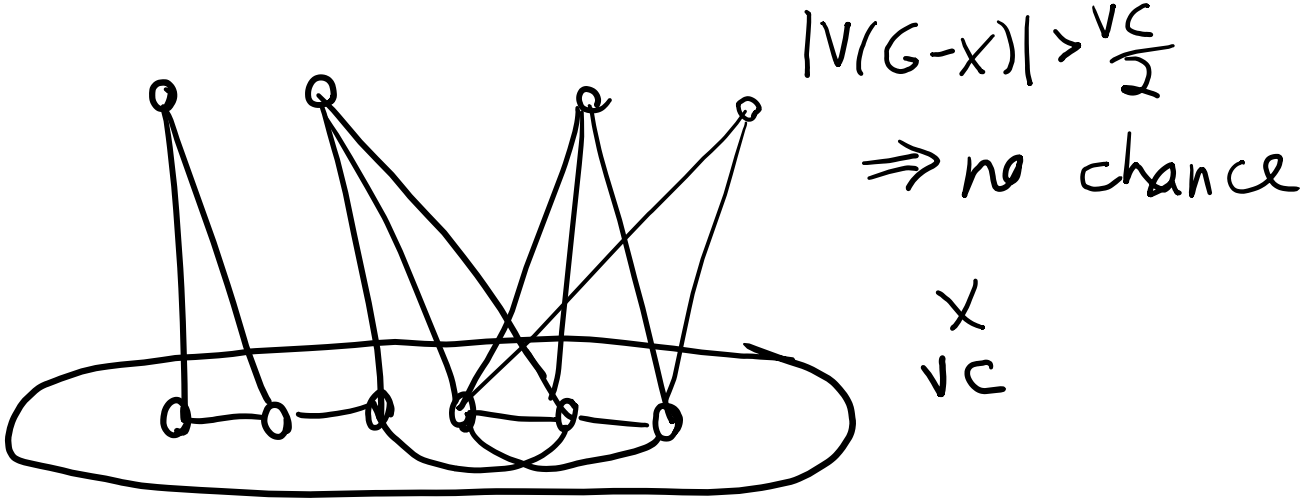


Triangle-Partition(vc): trivial $3/2 \cdot vc$ kernel.

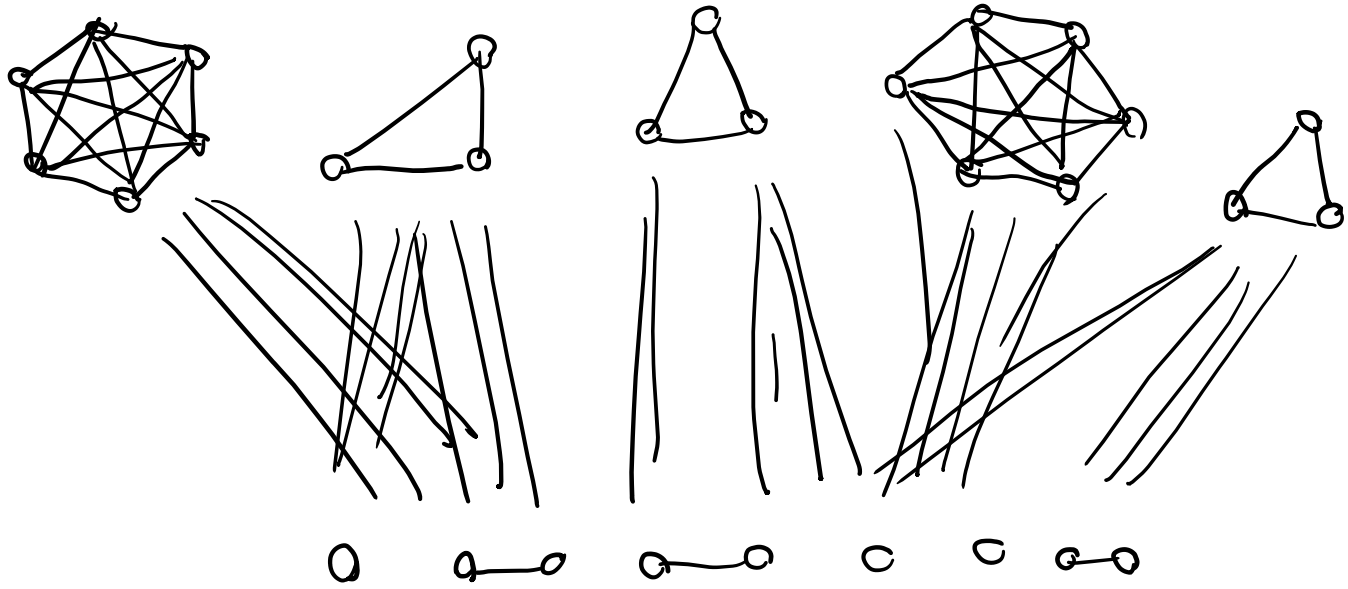
Let X be a vertex cover of G of size vc. Then $G - X$ is an independent set. Each vertex of $G - X$ must be in a triangle with two vertices of X. Thus if $|V(G - X)| > vc/2$, we say NO, otherwise $|V(G)| = |X| + |V(G - X)| \leq 3/2vc$.

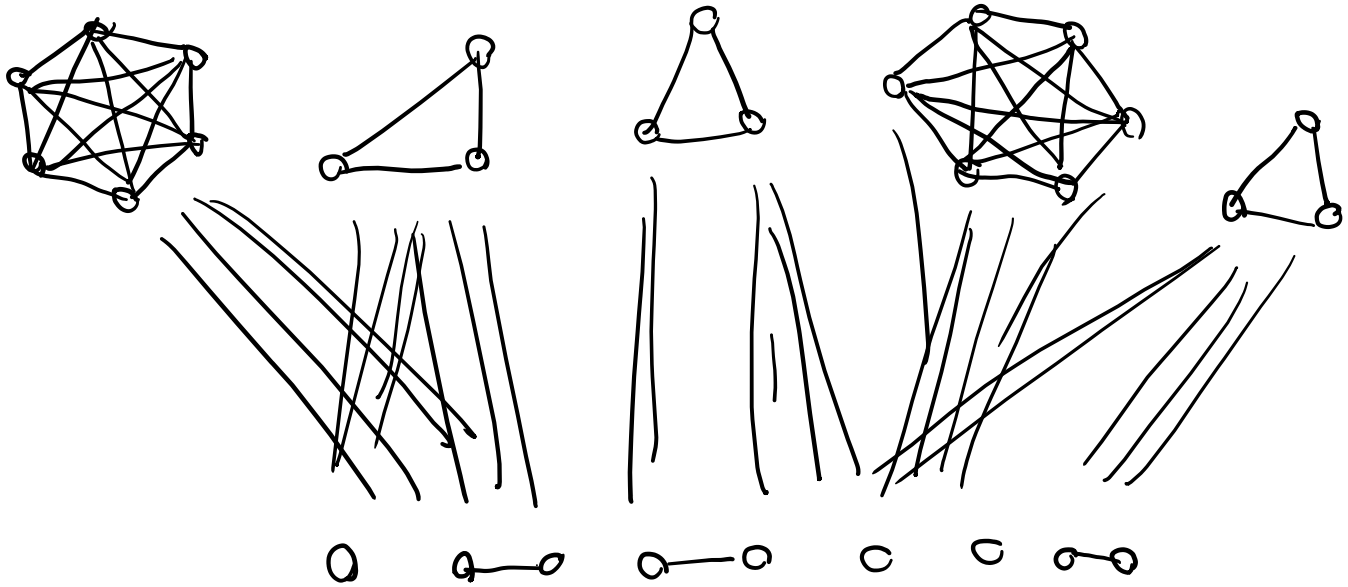


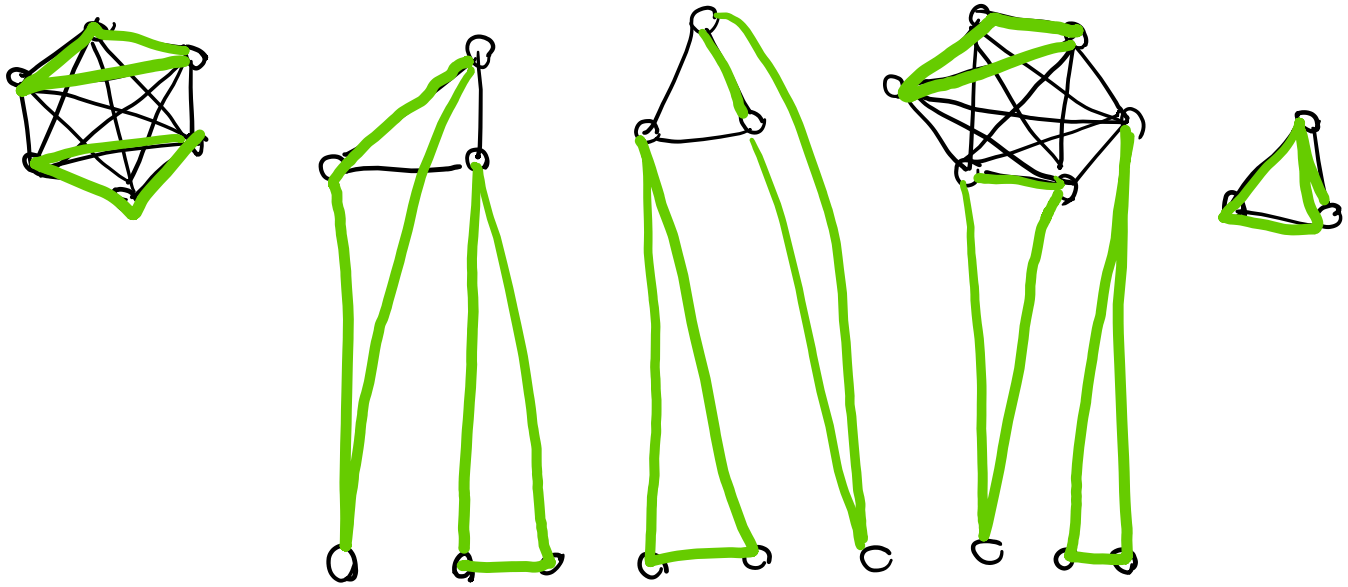
Triangle-Partition(tc): much, much less trivial.

Let $X \subseteq V(G)$ such that $G - X$ is a cluster graph. There can be many cliques in $G - X$ of size 0 (mod 3), that are "self-sufficient". But we don't know which ones are, and which ones use X.

Polynomial compression to a new problem called Conflict Free Assignment.







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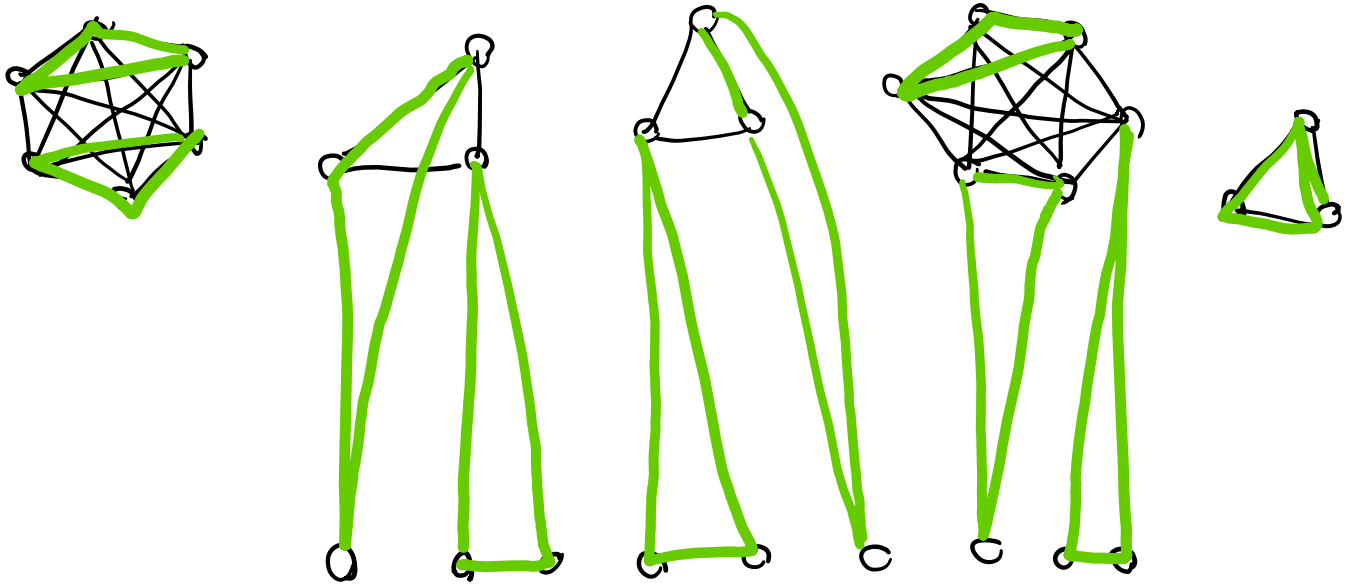


Conflict-Free Assignment

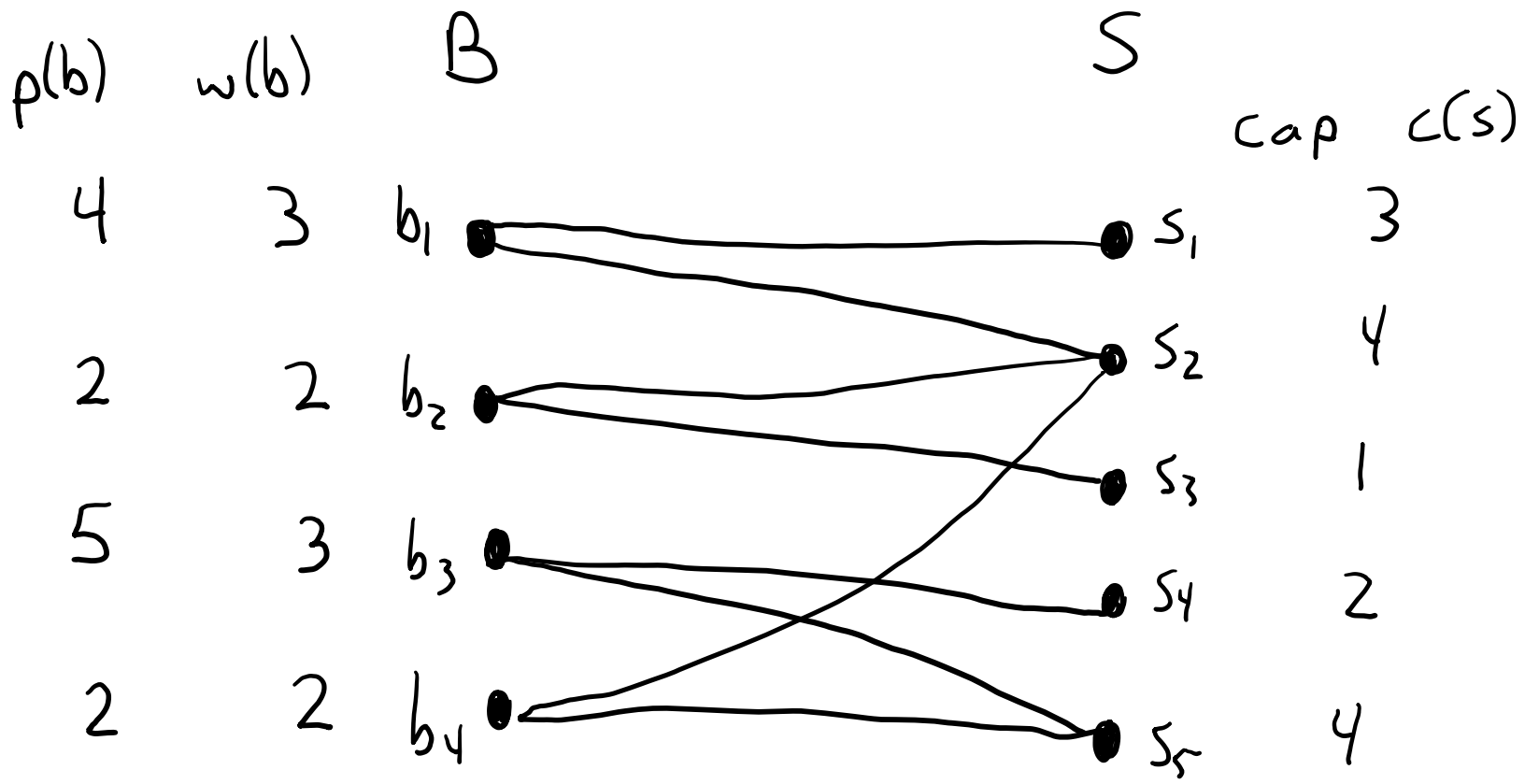
Input:

- bipartite graph $G = (B \cup S, E)$, $B =$ buyers $S =$ sellers.
- each buyer $b \in B$ has a weight $w(b)$ and a profit $p(b)$.
- each seller $s \in S$ has a capacity $c(s)$.
- $P \subseteq \binom{B}{2}$ forbidden pairs of buyers.

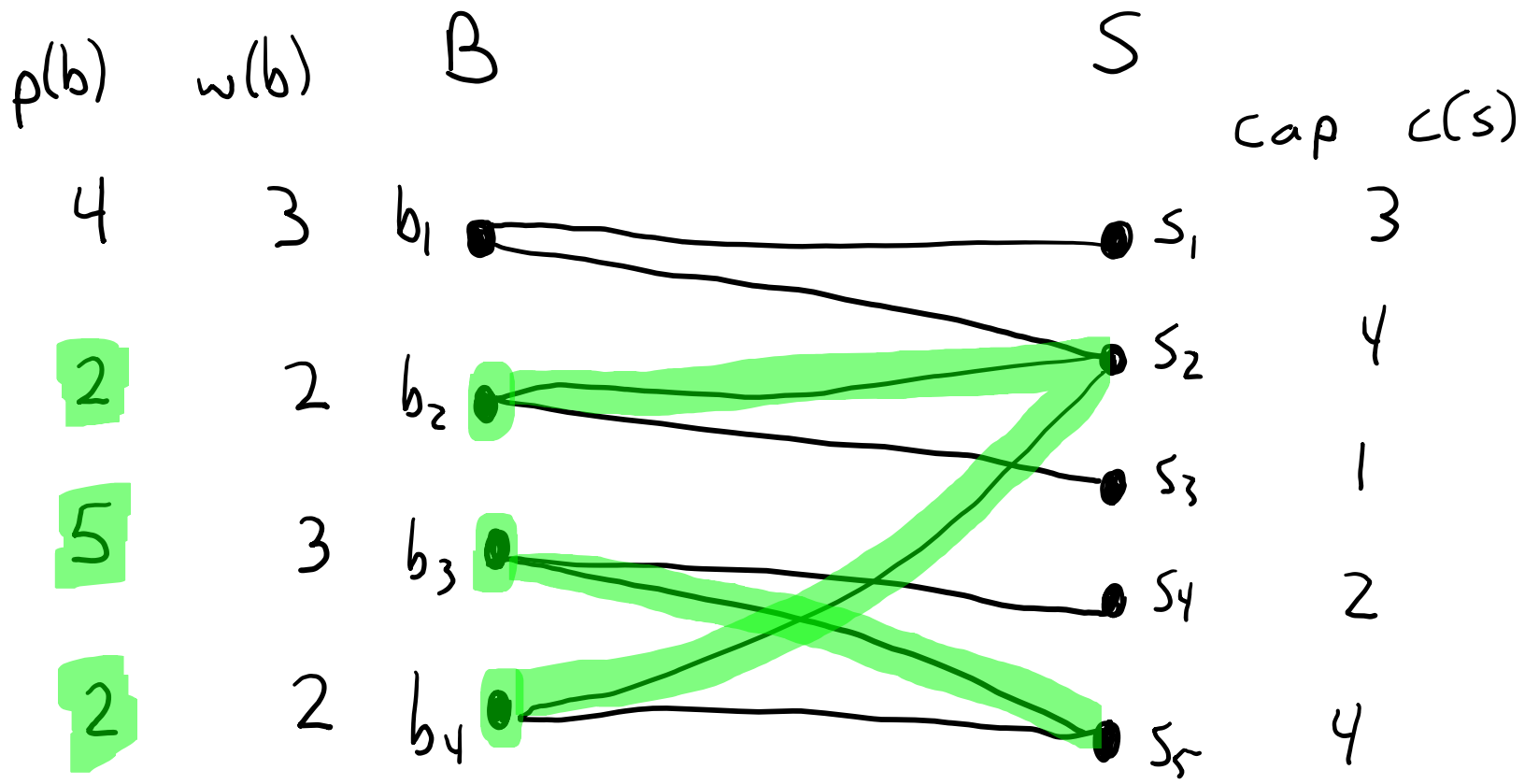
Output:

Partial assignment of buyers to sellers such that

- buyer b assigned to s only if $\{b, s\} \in E$.
- sum of weights $w(b)$ assigned to seller s is at most $c(s)$.
- if $\{b, b'\} \in P$, then one of b or b' is not assigned anywhere.
- sum of profit of assigned buyers is maximum.



$P = \{ \{b_1, b_3\} \}$



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Profit $= 2+5+2 = 9$

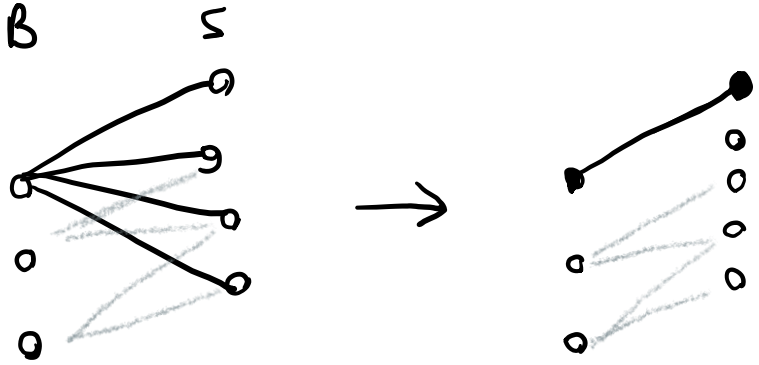
Theorem

The Conflict-Free Assignment problem admits a polynomial kernel in parameter $|B|$, the number of buyers.

Idea: if some buyer b has $\geq |B| + 1$ neighbors in S, there will be a seller available for b no matter what.

So, just remove all neighbors of b, add a new seller s_b whose sole neighbor is b.

As a result, each buyer has at most $|B|$ neighbors, hence we have a kernel of size $O(|B|^2)$.



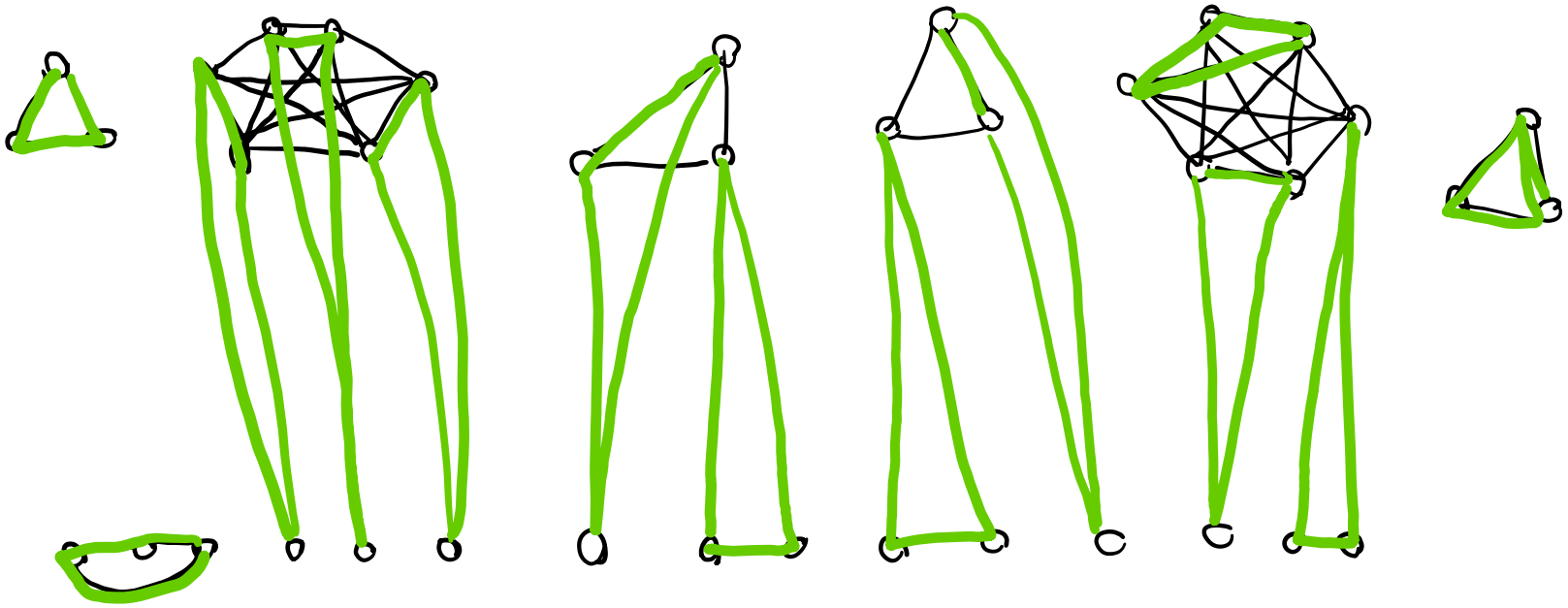
* Delete isolated vertices

Compression TrianglePartition(tc) to Conflict-Free Assignment

Assume that each clique of $G - X$ has size 0 (mod 3).

Then each clique "uses" 0 (mod 3) vertices of X to make triangles.

Challenge: assign triples of X to their rightful clique if any.



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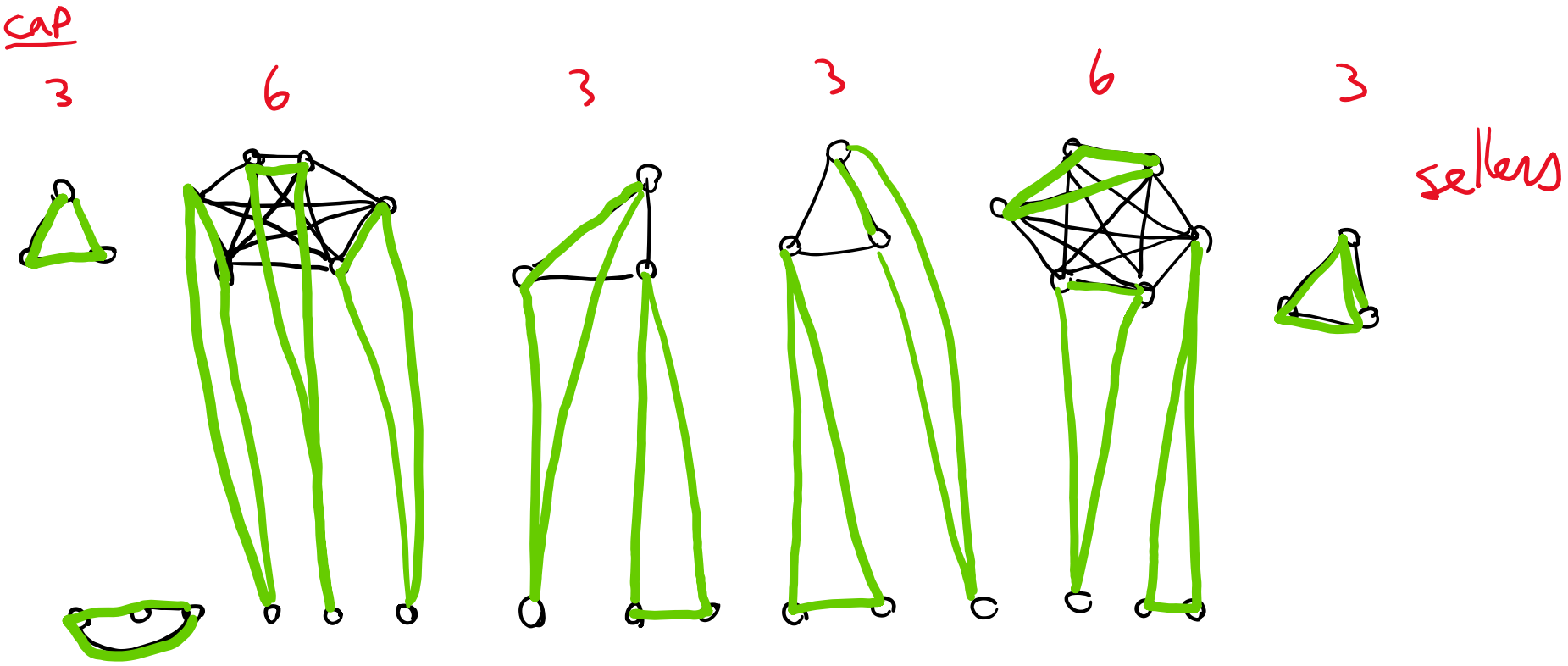
Then each clique "uses" 0 (mod 3) vertices of X to make triangles.

Challenge: assign triples of X to their rightful clique if any.



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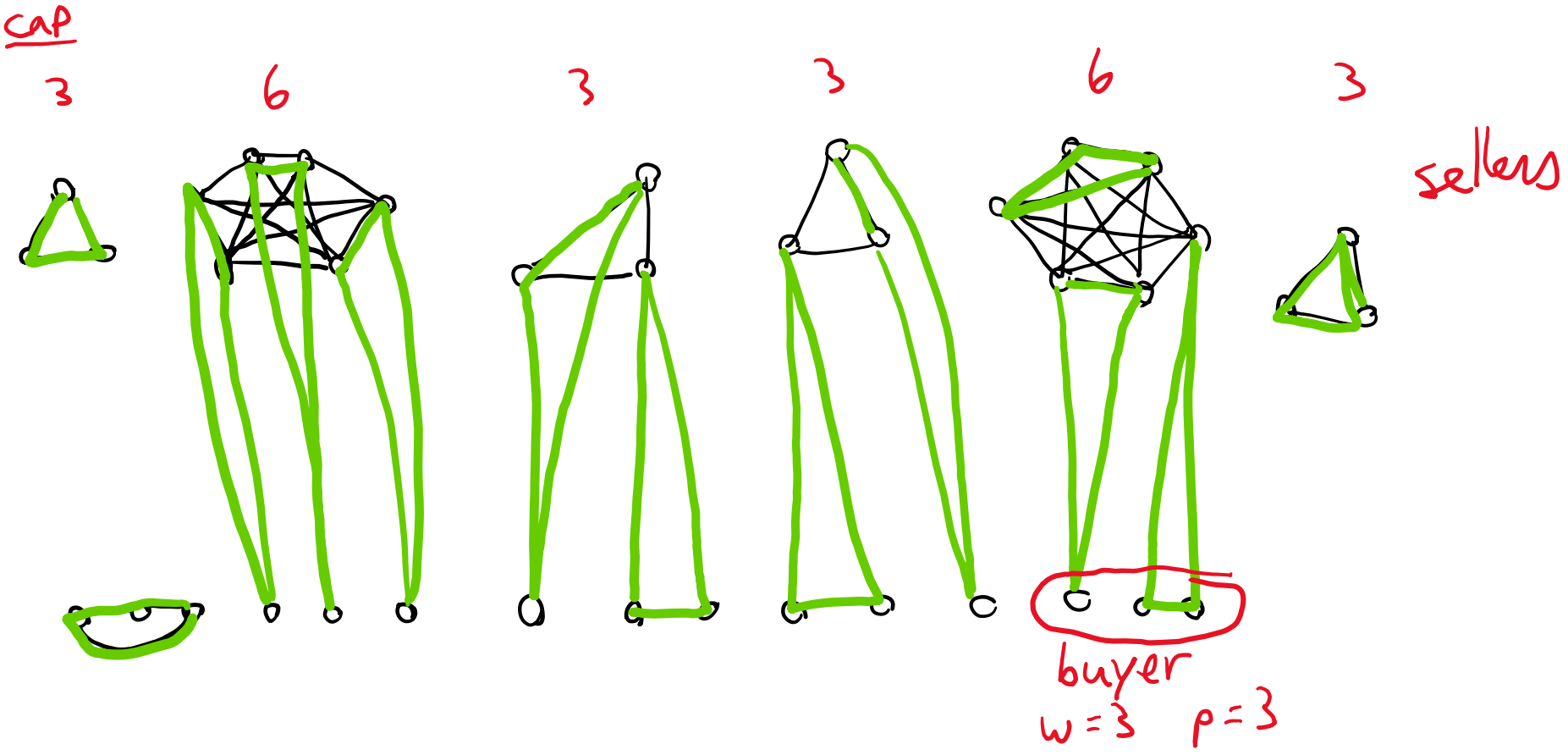

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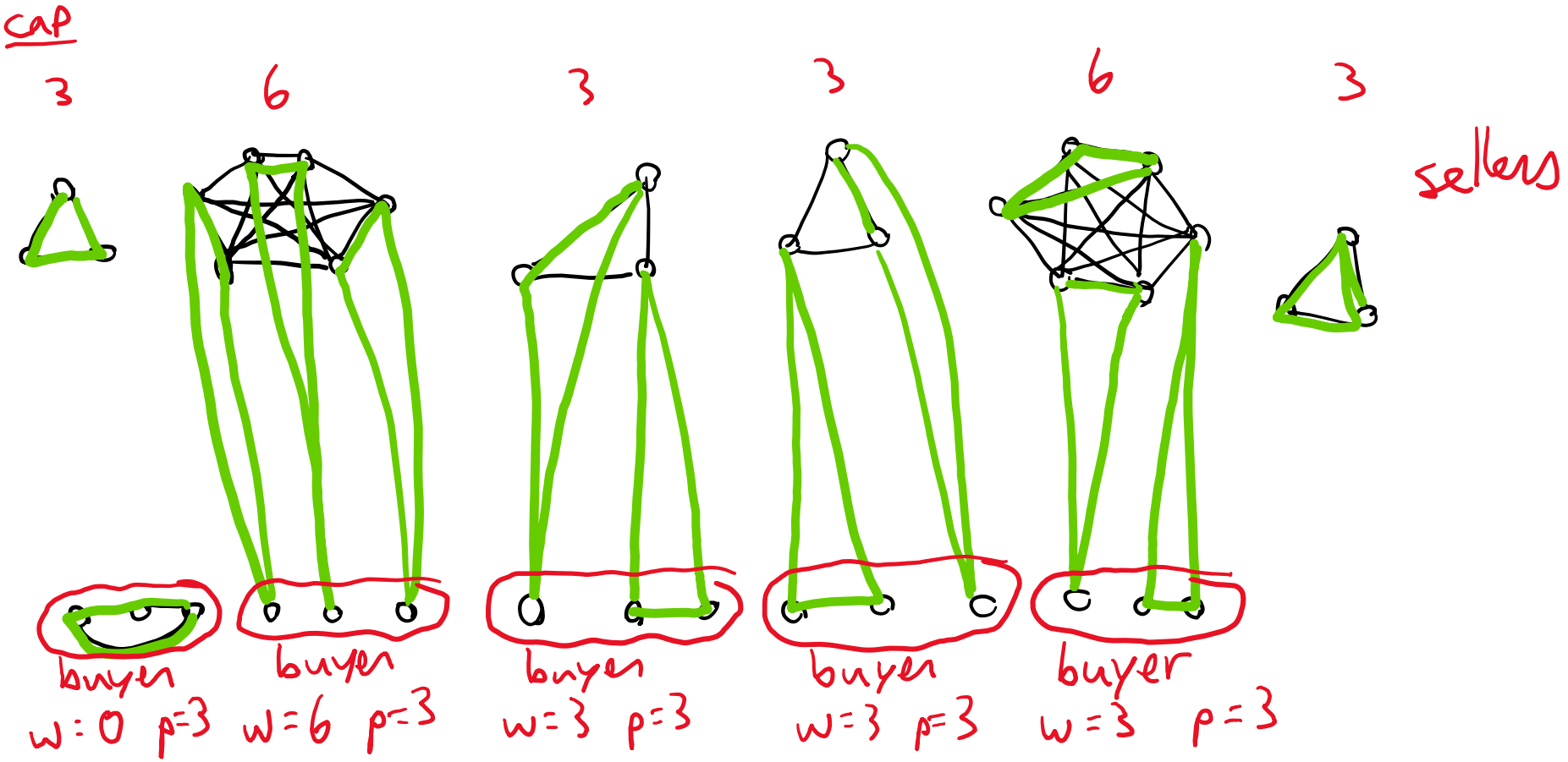

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Weight of triple = number of vertices it needs in a cluster to form triangles. (0, 3, or 6)

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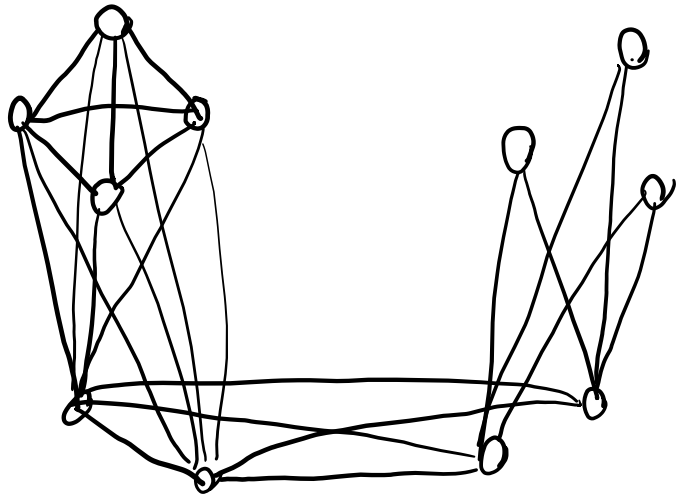
Neighborhood diversity - $nd(G)$

Introduced in [Lampis, 2010]

A *module* of G is a subset $M \subseteq V$ such that every vertex of M has the same neighborhood outside of M.

nd is the minimum size of a partition of $V(G)$ into modules that each induce a clique or an independent set.



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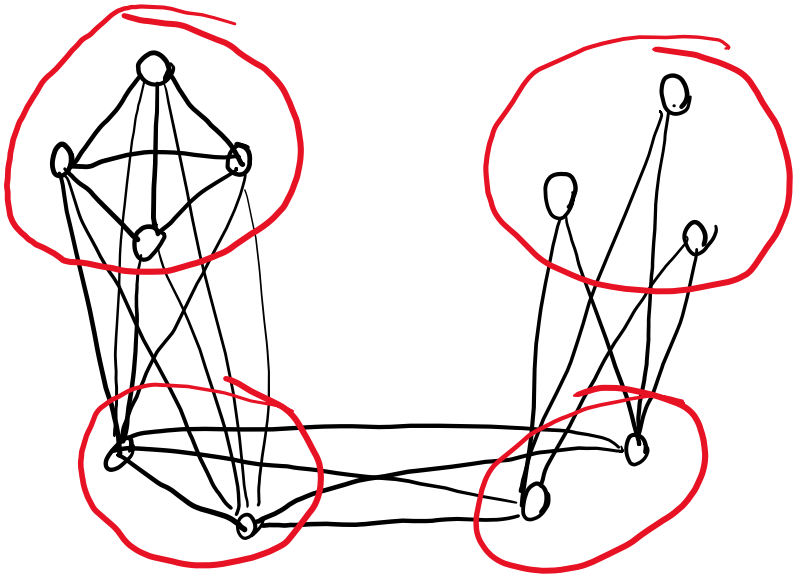

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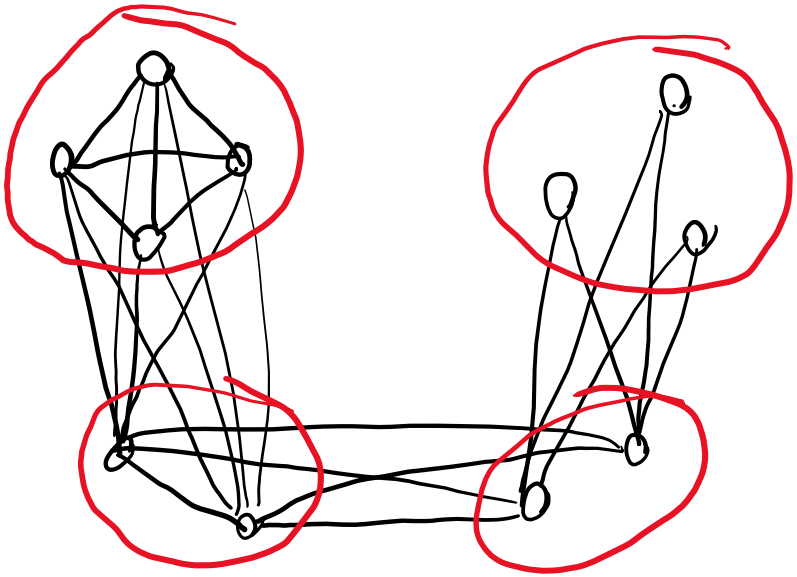
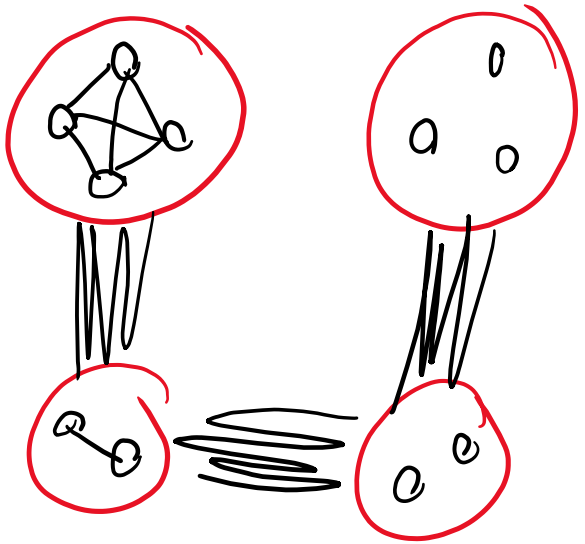


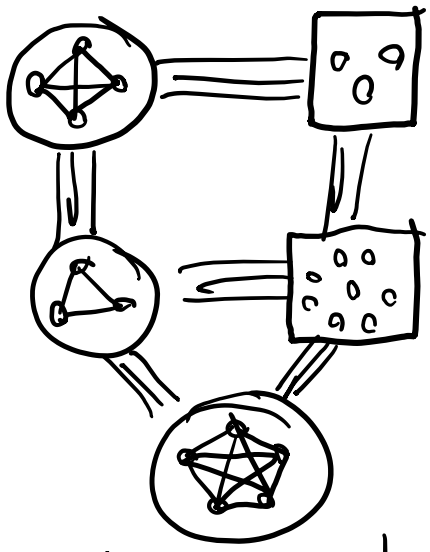


Kernels in nd

Proposition

If Q is a graph problem that can be solved in time $2^{O(nd^c)}$, then Q admits a polynomial compression.

Idea: if $nd^c < \log n$, solve in polytime $2^{O(nd^c)} = 2^{O(\log n)}$.
Assume that $nd^c \geq \log n$.



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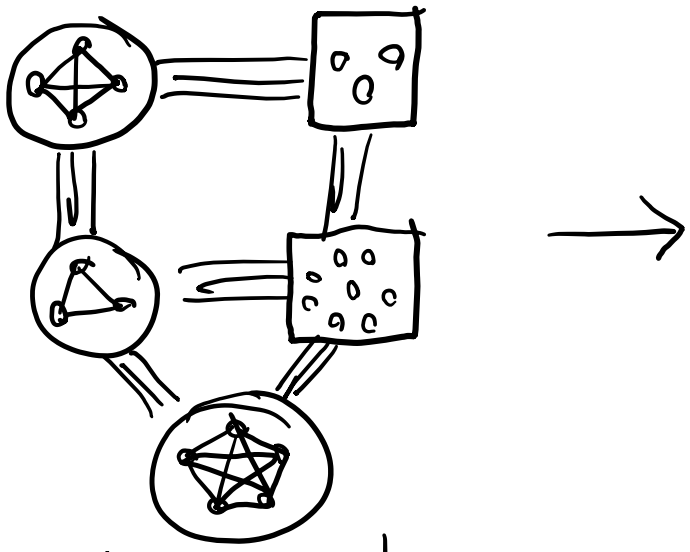

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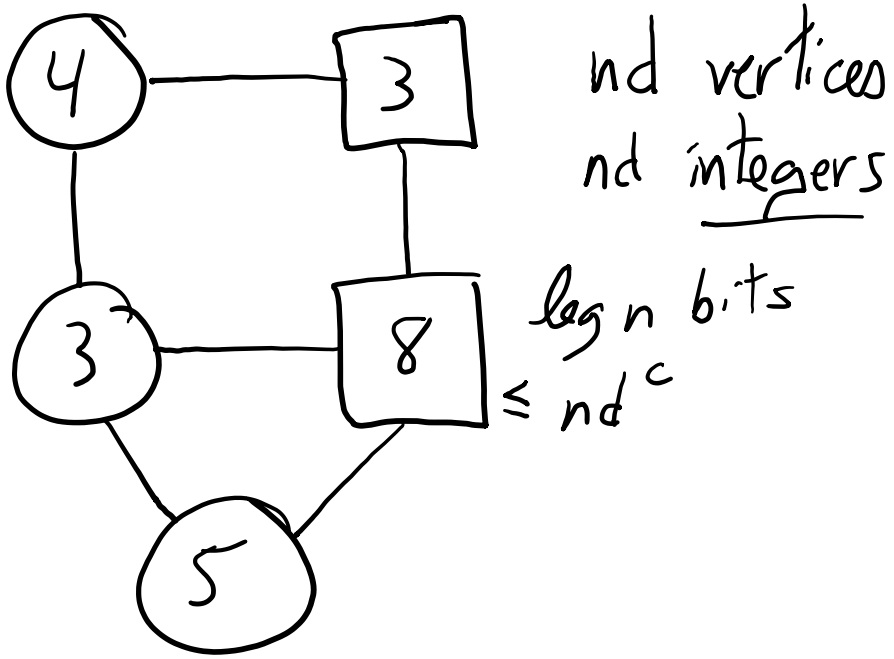

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For each clique module M, encode it as $(1, |M|)$.

For each ind. set. module M, encode it as $(0, |M|)$.

Represent graph with nd vertices, each encoded.

Total size is at most $nd \cdot \log n \leq nd^{c+1}$.

We have compressed into the problem

"same problem as Q but the size of the modules are given as integers".

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Since every graph property expressible in MSO_1 can be solved in $2^{O(\mathsf{nd})}|G|^{O(1)}$ [27], the result of Lemma 8 covers problems outside MSO_1. The following problem can be solved in $2^{O(\mathsf{nd})}n^{O(1)}$ time: Hamiltonian Cycle [1, 26, 27], Connected Vertex Cover [2], Connected Dominating Set [2], Steiner Tree [2], Dominating Set [7], Odd Cycle Transversal [29], Vertex Cover [14], Independent Set [14], Clique [14], Feedback Vertex Set [14], Induced Matching [14], and Chromatic Number [16], where the facts that $\mathsf{nd}(G) \leq \mathsf{cw}(G) + 1$ [27] and $\mathsf{mw}(G) \leq \mathsf{nd}(G)$ are needed when some results of the references are used. In addition, Partition Into Tirangle [24] can be solved in $O^*(2^{\mathsf{nd}^{O(1)}})$ time.

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Modular-width – $mw(G)$

A module of G is a $M \subseteq V$ such that every vertex of M has the same neighborhood outside of M.

∅, V, and every {v} for v ∈ V are called the trivial modules.

If all modules of G are trivial modules then G is a prime graph.

mw is the number of vertices in the largest prime induced subgraph of G.

* mw = max of CCs

Chromatic number(mw)

Proposition

Chromatic number admits no polynomial compression in parameter mw, unless coNP $\subseteq$ NP$_{/poly}$.

AND cross composition

A problem $L \subseteq \Sigma^*$ AND-cross-composes into parameterized problem $Q \subseteq (\Sigma^* \times N)$ if, given t instances $x_1, \ldots, x_t$ of the same size n, one can output in polytime in tn an instance (I, k) of Q such that:

- $(I, k) \in Q$ if and only if $x_i \in Q$ for each $i = 1, \ldots, t$
- $k \leq n + \log t$

Theorem [Bodlaender, Jansen, Kratsch, 2014]

If an NP-hard problem L AND-cross-composes into Q, then Q has no polynomial compression, unless coNP $\subseteq$ NP$_{/\text{poly}}$.

Definition 7 (and-cross-composition)

Let $L \subseteq \Sigma^*$ be a set and let $Q \subseteq \Sigma^* \times \mathbb{N}$ be a parameterized problem. We say that L *and-cross-composes* into Q if there is a polynomial equivalence relation R and an algorithm which, given t strings $x_1, \ldots, x_t$ belonging to the same equivalence class of R, computes an instance $(y, k) \in \Sigma^* \times \mathbb{N}$ in time polynomial in $\sum_{i=1}^{t} |x_i|$ such that:

1. the instance (y, k) is yes for Q iff all instances x_i are yes for L;
2. k is bounded by a polynomial in $\max_{i=1}^{t} |x_i| + \log t$.

Theorem 8

Let L be an NP-hard problem under Karp reductions. If L and-cross-composes into a parameterized problem Q, then Q does not admit a PC unless NP $\subseteq$ coNP/poly.

Chromatic number(*mw*)

Proposition

Chromatic number admits no polynomial compression in parameter *mw*, unless coNP $\subseteq$ NP$_{/poly}$.

AND-cross-composition from (unparameterized) Chromatic Number.

Chromatic number(mw)

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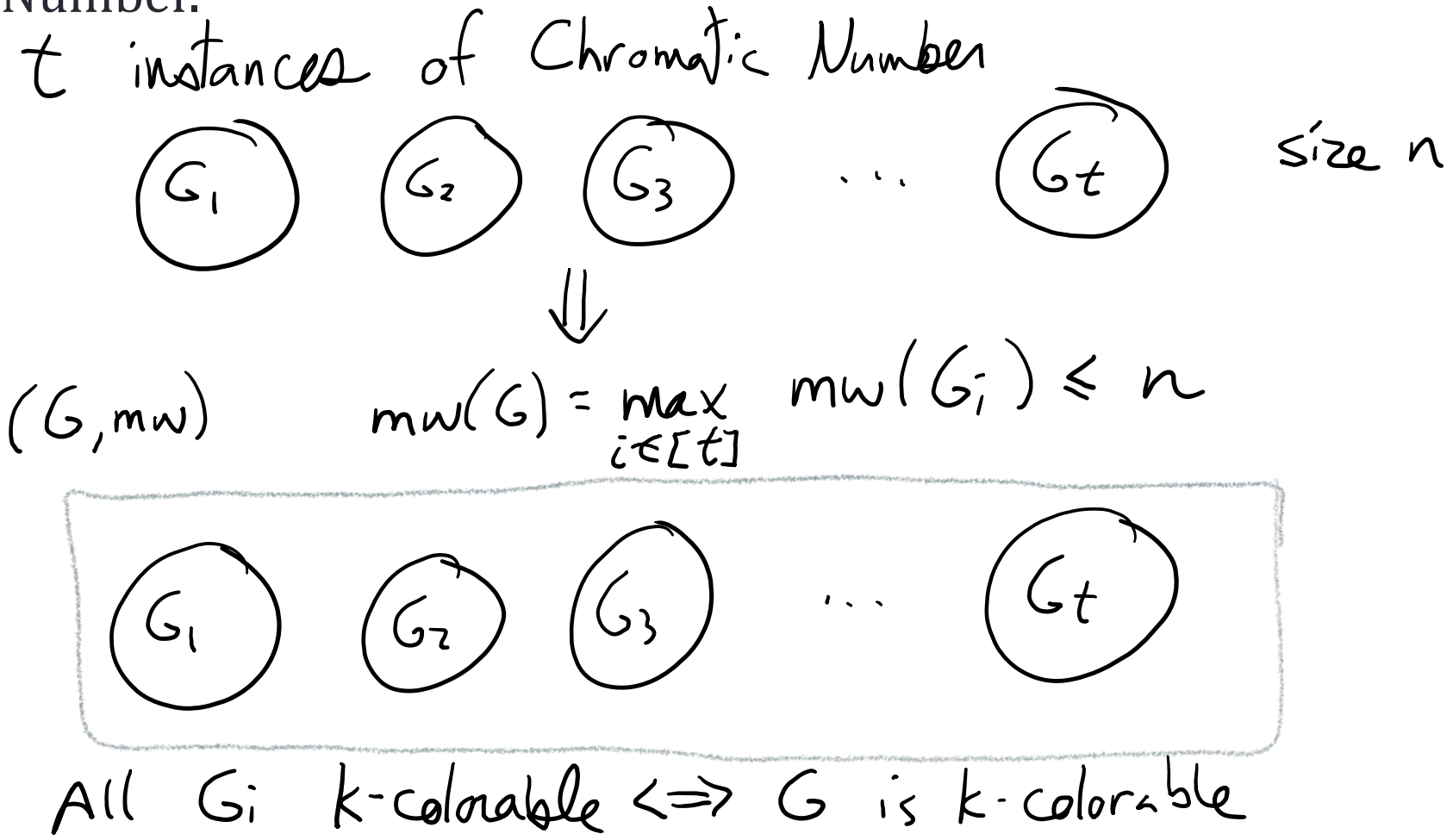

Conclusion

- Many established techniques used in this paper
  - Reduction rules, MK-hardness reductions, compression, Turing kernels, cross-composition, nd encoding and MSO, ...
- Are there problems in structural parameterization that will require new techniques?
- Other structural parameters of interest?
  - feedback vertex set, treewidth, ...
- Meta-theorems for kernelization under structural parameters? (e.g. similar to the nd lemma)